Technical Comments

Aerodynamic Loads on Bluff Bodies at Low Speeds

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THE wealth of information on low-speed buffet loads presented during the first session† on Structural Dynamics of the AIAA Fifth Annual Structures and Materials Conference in Palm Springs, Calif., April 1–3, 1964, was very impressive and educational.[1–3] Yet it appears that much remains to be understood, even qualitatively. The following qualitative observations emphasize those aspects of the problem which seem important for such increased clarification to an exelastic aerodynamicist, largely turbulent and transitional, as well as nonlinear. They are offered in a constructive spirit, for here friendly teamwork between "aeros" and "elasts" is truly in order.

1) It is known[4] that the effects of very small roughness on large structures, such as that of a 0.003-in. tape (0.0002 to 0.0004 diam) stretched along cylinder generators, are very large and overpower substantial variations of structural parameters. The airloads on a cantilever cylinder with a hemispherical tip are changed by an order of magnitude when the hemispherical tip is cut off flat or when it is rimmed with a cardinal-hat brim. These are only samples of the documented, seemingly erratic behavior of cantilevers in high Reynolds number winds. An important clue is the occurrence of large effects as a result of very small changes. Such behavior carries the earmarks of instability phenomena or more likely of a chain of interacting instabilities.

2) One likely source of multiple instabilities is the surfaces of concentrated vorticity which envelop the body first in the form of a boundary layer and then as free vortex sheets, increasingly more corrugated as Reynolds number rises. Of the several possible alternate viewpoints, emphasis on vorticity also holds promise because of its classical relationship to loads: lift and drag. Furthermore, in the immediate vicinity of the body where the loads arise, the Helmholtz vorticity laws should be valid, because the viscous dissipation in the free vortex surfaces is insignificant even in presence of partial randomness. The instantaneous law for induction of velocity in terms of vorticity also remains valid at the low subsonic Mach numbers.

3) Our present knowledge of the instabilities associated with the vorticity blanket, namely, a) transition[5,6] of a boundary layer or free vortex sheet to turbulence, b) separation of boundary layers, c) interaction and self-interaction of separated vortex surfaces, laminar or turbulent, and d) possible local reattachment of separated boundary layers, tells us that, unless controlled by strong uniformizing influences, these phenomena tend to be highly localized in time and space, i.e., unsteady and three-dimensional, so that when cascaded, any orderliness disappears or is hidden behind a random-like modulation. However, our present knowledge of items a–c also tells us that such aerodynamic instabilities are "soft," that is, easily influenceable in the early stages of their development.

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† These "prepared comments" were given at the end of the session.

4) Plane acoustic waves‡ and vibrations of essentially two-dimensional objects such as wires or ribbons, addition of thinnest strips of tapes, etc., are known to have strong two-dimensionalizing influence on instabilities a–c of Sec. 3. The space-time correlations of the subsequent turbulent motions are strongly enhanced. This is especially so for the larger-scale, lower-frequency motions commensurate to the characteristic scales of the boundaries. Turbulent motion on such scales is rather immune to the homogenizing process of the so-called cascading turbulent interaction and continues to carry the more orderly imprint of its genesis. In other words, nonrandomly generated turbulent motion is unlikely to be fully random§ in its larger components, especially in its early history (near-wake).

5) Long, essentially two-dimensional tapes of fixed small thickness undoubtedly can exercise a spanwise organizing influence¶ in some ranges of Reynolds number, depending upon boundary-layer thickness and upon whether their primary influence is on the transition or the separation phenomenon (hence, also the sensitivity to position on the body). Oscillations of flexible launch vehicles also provide a coherent perturbation of the unstable vortex sheets around the vehicle and could decrease the randomness of the local flow field should the oscillatory amplitudes become sufficiently large. One may call the possible influence of the structural oscillations "feedback," or "coupling," or "negative damping,"

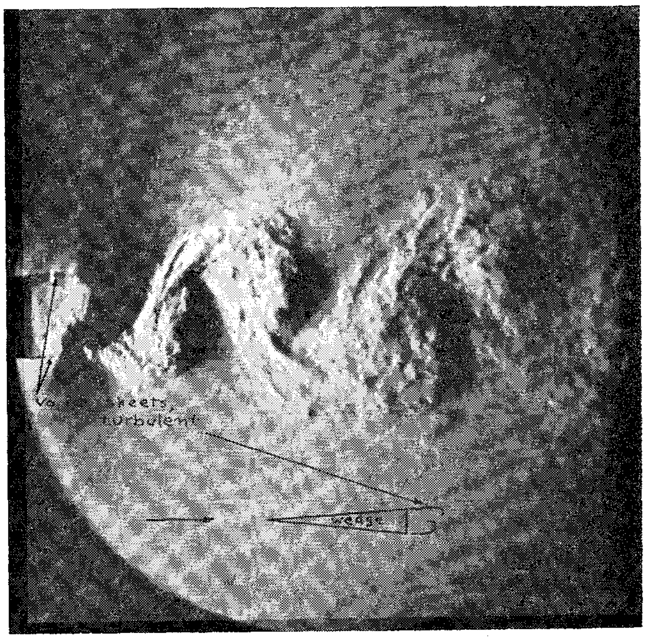

Fig. 1 Schlieren of unsteady flow past the base of a 15-cm-long two-dimensional wedge with half angle of 4.75° and turbulent boundary layers (courtesy of H. Thomann and Aeronautical Research Institute of Sweden).

‡ It is not intended to suggest that sound is important in the present context. The example drives home the sensitivity of vortex-sheet instability to weak but organized influences.

§ This suggests that we should pay attention to the principle of "impure chaos" on which the proven statistical system of the mathematician Thorp for beating the dealer in blackjack is based.[9]

¶ In the words of A. Gerald Rainey at this session, "promote an increased degree of togetherness."

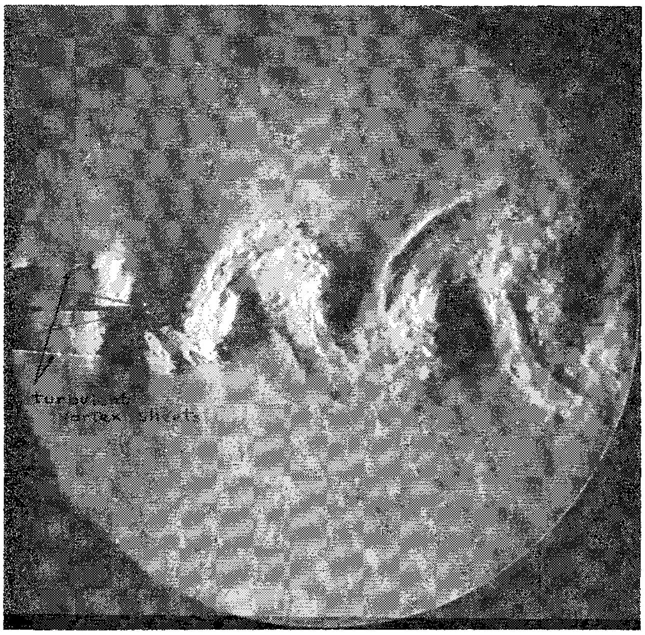

Fig. 2 Schlieren of unsteady flow past the base of a 15-cm-long two-dimensional wedge with half angle of 4.75° and turbulent boundary layers, with a 4-cm full-span splitter plate.

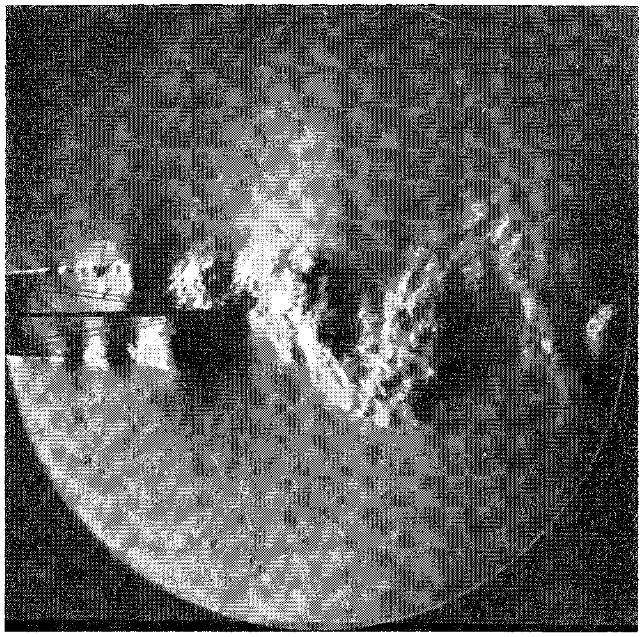

Fig. 3 Schlieren of unsteady flow past the base of a 15-cm-long two-dimensional wedge with half angle of 4.75° and turbulent boundary layers, with a 6-cm full-span splitter plate.

weak or strong, depending upon one's point of view and background. From the probable role of the chain of the relevent aerodynamic instabilities, one could expect a) essentially no influence for very rigid structures; b) an increasing amplitude-dependent influence as the effect of the oscillations becomes comparable to the other inputs into the instability mill; and c) a possible saturation limit of the oscillation influence if the large amplitude should induce all the coherence of which the nonlinear phenomenon is capable.

It is believed that a nondogmatic point of view, which allows not only for randomly driven systems, or for true self-excitations, but also for more subtle partially organizing influences, is necessary to explain the oscillations of flexible cantilevers across the Reynolds number range. For instance, we should not a priori reject the possibility that the motion of the aforementioned hemispherical tip would a) influence the local three-dimensional separation of the boundary layer and its unstable rollup into streamwise "tip vortices" and b) influence the time-dependent relative position of the tip vortices with respect to the body surface and hence the time variations of the induced load.

6) It has often been said that once the boundary layer on an elongated body with a bluff base becomes turbulent even its near wake becomes incoherent and the loads on the body can be treated as random. Roshko's high Reynolds number hot-wire measurements[7] at a single point, 7.3 diam downstream of a circular cylinder, indicate that such generalizations may be dangerous.

A pictorial demonstration of an example of such coherent-wake oscillations when the separating boundary layer is definitely turbulent may be instructive and should clarify some of the preceding concepts. The following beautiful schlieren pictures[8] of a turbulent wake behind an elongated wedge are due to Hans Thomann of the Aeronautical Research Institute of Sweden. The characteristics of the symmetric wedge spanning the tunnel are: 4.75° half angle; 15-cm chord, 2.5-cm base height; Reynolds number (base height) = $2.7.10^5$; Re(boundary-layer run) = $1.7.10^6$ with consequently turbulent boundary layer; Mach number = 0.556 with local velocities well below sonic. The base of the triangle is just visible on the left of the figures, as are the full-span splitter plates (strengthened against vibration by diagonal wire supports, which obstruct the view but not the flow). The lengths of the splitter plates are 0, 4, 6, and 12 cm, i.e., 0, 1.6, 2.4, and 4.8 base heights in Figs. 1–4, respectively.

There are three cooperative reasons for the enhanced coherence of this particular turbulent wake, which is undoubtedly random as far as the smaller scale, high-frequency motion is concerned:

a) There is a sharp trailing edge with a very strong two-dimensionalizing effect, especially when compared to circular cylinders.

b) The separated turbulent shear layers (arrows in Figs. 1 and 2) are relatively thin with respect to the lateral interaction distance between them. Hence the local distribution of vorticity is secondary, and it is the net vorticity across the layers which dominates. Notice that as the longer split-

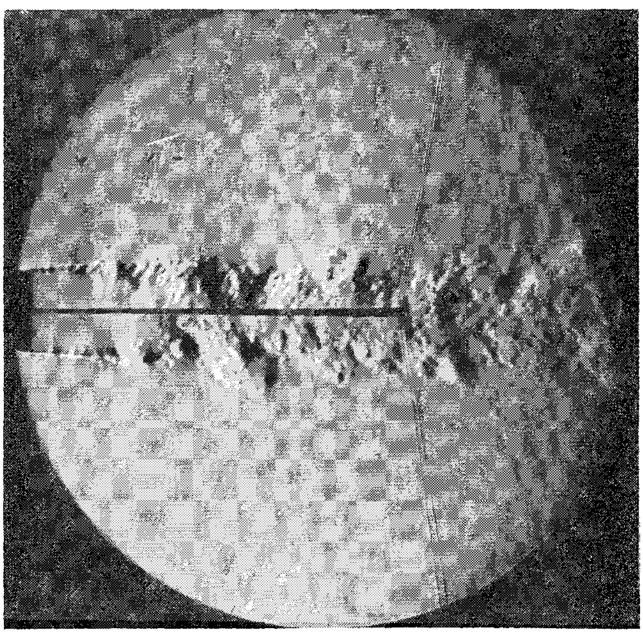

Fig. 4 Schlieren of unsteady flow past the base of a 15-cm-long two-dimensional wedge with half angle of 4.75° and turbulent boundary layers, with a 12-cm full-span splitter plate.

ter plates translate the interaction region between the layers away from the two-dimensionalizing influence of the sharp shoulders, and as the vortex layers get thicker relative to the lateral interaction distance, and more contorted, the subsequent motion gets more and more incoherent in accordance with our expectation. The local oscillating pressures on the base are also undoubtedly relieved.

c) There is a confining influence because of the sidewalls of the tunnel, the span being only three times the base height. (Although the interaction in the side-wall boundary layers brings about some three-dimensional effects, the confining, two-dimensionalizing effect is thought to be more important at these Reynolds numbers.) The "softness" of the instability (Sec. 3) is reflected in the manner in which the changing boundary conditions influence the dimensionless frequency (multiplied by base height and divided by freestream speed). These so-called Strouhal numbers are 0.27, 0.30, and 0.24 in Figs. 1–3, respectively.

Roshko's phenomenon of the recovery of pronounced perodicity in the wake[7] of two-dimensional circular cylinders at "transcritical" Reynolds numbers could be more or less similar to state of affairs in Figs. 2 or 3, the splitter plates simulating the partial obstruction due to the circular base. In Roshko's case, an increase in coherence** is apparently due to the complete disappearance of several links in the instability chain, namely, of the laminar (or mixed laminar-turbulent) separation some 80°–90° from the front stagnation line, of the instability and/or transition of the separated layer, and of the consequent turbulent or mixed reattachment of the layer, which are present in the more random flows for an indefinite range of Reynolds number above approximately 200,000 on smooth circular cylinders. At Reynolds numbers past 3.5×10^6, the universal transition to turbulence upstream of the 80° station and the single turbulent separation in presence of a strong, fairly two-dimensional, adverse pressure gradient could be roughly comparable to the sequence of instabilities in Figs. 2 or 3. Again, we note that the turbulence of Roshko's boundary layers does not necessarily bring about complete randomness.

7) In conclusion, the writer feels that increased attention to the behavior of the whole vorticity envelope, and in particular to the subtle small influences (including that of the organizing motion of the body) on its multiple instabilities, should help to understand the less random aspects of these cases of "impure chaos."

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** Low freestream turbulence may have also helped. It is also possible that increased organization of wakes may occur only locally over spanwise segments of the cylinder and wake. Another possibility can be sketched in terms of the flow downstream of the right rim of Fig. 4: This nearly homogenized wake could undergo large-scale inviscid instability in its own right. Clearly, these many possibilities indicate the need for increased aerodynamic information in conjunction with structural load testing.

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Strain-Displacement Relations in Large Displacement Theory of Shells

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Nomenclature

A_1, A_2	= functions of α_1, α_2
$B_1, B_2, \ldots B_6$	= functions of u, v, w
$C_1, C_2, \ldots C_6$	= functions of $\bar{u}, \bar{v}, \bar{w}$
$D_1, D_2, \ldots D_6$	= functions of θ, ψ, χ
H_1, H_2, H_3	= Lamé coefficients
R_1, R_2	= principal radii of curvature
X, Y, Z	= right-hand Cartesian coordinate system
u, v, w	= displacement components along the α_1, α_2, z directions, respectively, at an arbitrary point
$\bar{u}, \bar{v}, \bar{w}$	= displacement components along the α_1, α_2, z directions, respectively, on middle surface of shell
z	= coordinate in the direction of the normal to the shell surface
α_1, α_2	= curvilinear coordinate lines that are also lines of principal curvature of middle surface of shell
$\alpha_{31}, \alpha_{32}, \alpha_{33}$	= functions of $\bar{u}, \bar{v}, \bar{w}$
$\epsilon_{11}, \epsilon_{22}, \epsilon_{zz}, \epsilon_{12}, \epsilon_{1z}, \epsilon_{2z}$	= normal and shear strains at an arbitrary point
$\bar{\epsilon}_{11}, \bar{\epsilon}_{22}, \bar{\epsilon}_{12}$	= normal and shear strains on middle surface of shell
θ	= function of $\bar{u}, \bar{v}, \bar{w}$
$\nu_{11}, \nu_{22}, \nu_{12}$	= functions of $\bar{u}, \bar{v}, \bar{w}$
$\chi, \chi_{11}, \chi_{22}, \chi_{12}$	= functions of $\bar{u}, \bar{v}, \bar{w}$
ψ	= function of $\bar{u}, \bar{v}, \bar{w}$

I. Introduction

STRAIN-DISPLACEMENT relationships are one of the most important sets of formulas in structural analysis. An incorrect relationship will give rise to errors in the results and conclusions of various structural analyses, such as buckling, vibrations, or stress analysis. Using the large displacement theory and Kirchhoff's assumption on the preservation of the normal element, Ref. 1 derived a set of strain-displacement relationships. These relationships can be improved by 1) correction of an error made in the derivation and 2) clarification of the sign convention for the principal radii.

II. Analysis

Let the middle surface of the shell be defined by

$$X = X(\alpha_1, \alpha_2) \qquad Y = Y(\alpha_1, \alpha_2)$$
$$Z = Z(\alpha_1, \alpha_2)$$

where X, Y, Z are rectangular coordinates, and α_1, α_2 are

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